

Multiplicative Reasoning

MATHEMATICS KEY STAGE 4

Introduction

SUMMARY:

Use compound measures to understand how savings and loans work.

STRUCTURE:

The content is divided into two (approximately) 60 minute lessons.

PRESENTATION:

Italicised text are suggested scripts for the teacher to say. There are explanatory notes to aid quick understanding of some of the finance material.

GCSE ASSESSMENT OBJECTIVES ADDRESSED IN THE LESSON¹

The mathematical content specifications in this presentation are those used in the Mathematics GCSE Subject content and assessment objectives and are identified in red.

- R1:** change freely between related standard units (eg, time, length, area, volume/capacity, mass) and compound units (eg, speed, rates of pay, prices, density, pressure) in numerical and algebraic contexts
- R6:** express a multiplicative relationship between two quantities as a ratio or a fraction
- R16:** set up, solve and interpret the answers in growth and decay problems, including compound interest and work with general iterative processes

¹ The expectation is that:

- All pupils will develop confidence and competence with the content identified by standard type
- All pupils will be assessed on the content identified by the standard and the underlined type; more highly attaining pupils will develop confidence and competence with all of this content
- Only the more highly attaining pupils will be assessed on the content identified by **bold** type. The highest attaining pupils will develop confidence and competence with the bold content.

Before lesson 1



HOMEWORK A FOR THE PUPILS

Imagine that you want to save some money for a car. Let's say that the car would cost £5,000 and that you already have savings of £1,500.

Using the website below, answer all the following questions:

1. Using interest rates of 1%, 3% and 5%, find out how long it will take you to save the amount needed for the car by saving £50 each month.
2. Using interest rates of 1%, 3% and 5%, find out how much you would need to save if you needed to buy the car in two years.

The website is: <https://www.moneyadviceservice.org.uk/en/tools/savings-calculator>



FOR THE TEACHER

The website is from the Money Advice Service, a body set up by the government. The two questions correspond to the two calculation links that the pupils will see on the first page after they click on the link.

The objective of the task is to give the pupils a reality check on the interest rates available. Interest rates are very low and are likely to be less than 1% and, so, this will increase the amount of time required to save for the car (for the first question) and increase the amount of money needed to be put aside (for the second question).

Review the submitted homework before the lesson and select one or two to discuss in class (detailed in an activity below).

Lesson time

GENERAL NOTES

The associated powerpoint presentation has supporting script notes to help you. These can best be viewed by clicking 'View/Notes Page' in powerpoint. Items in *italics* are a proposed script for you to say. The content of the lesson plan follows closely the notes contained in the powerpoint.

Teaching and activities

CLASS DISCUSSION: INTRODUCE DISCUSSION ON THE HOMEWORK

Ask for volunteers to say or present their homework results to the class. (How this is tackled by the teacher will vary depending on how willing the class is to present their ideas). It may be preferable to have group ideas presented or to choose one or two pieces of homework for the teacher to present.



EXPLANATORY NOTES

A key skill being developed is using financial websites confidently and discovering that there is useful information and web content available.



HOMEWORK FOR THE PUPILS: THE FOLLOWING WORK TO BE SET BEFORE THE LESSON:

Imagine that you want to save some money for a car. Let's say that the car would cost £5,000 and that you already have savings of £1,500.

Using the website below, answer all the following questions:

1. Using interest rates of 1%, 3% and 5%, find out how long it will take you to save the amount needed for the car by saving £50 each month.
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The website is: <https://www.moneyadviceservice.org.uk/en/tools/savings-calculator>

POINTS FOR THE TEACHER TO WATCH-OUT FOR:

1. Interest accumulation on savings (and loans for that matter) is determined, in a large way, by the amount of time that savings are held. While there are changes to the answers to question 1 as interest rates change, you will see that interest rates have a smaller and smaller effect as the length of time of the savings period is reduced. Perhaps this could be demonstrated to pupils in class. The essential mathematical idea is that compounding needs compounding periods to have a big effect (unless interest rates are unrealistically high).
2. Interest rates on savings are very low. While 1% is achievable, 3% and certainly 5% are not. Also, the interest rates used do not take account of tax (they are gross rates without tax being deducted). Interest on savings accounts (unless you are a child) have tax deducted at 20%. This is likely to be an important factor for a person buying a car who will almost certainly be working - and a taxpayer - in order to buy the car and pay for its running costs.

THE ANSWERS TO THE QUESTIONS ARE:

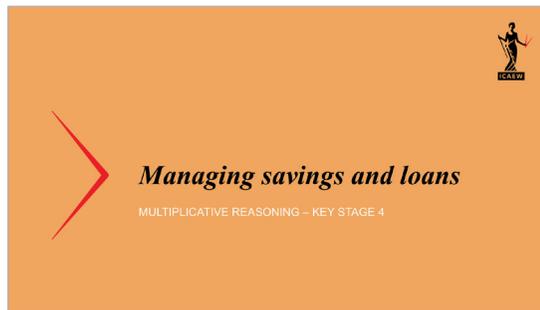
Question 1

- 1% 5 years 7 months
- 2% 5 years 1 month
- 5% 4 years 8 months

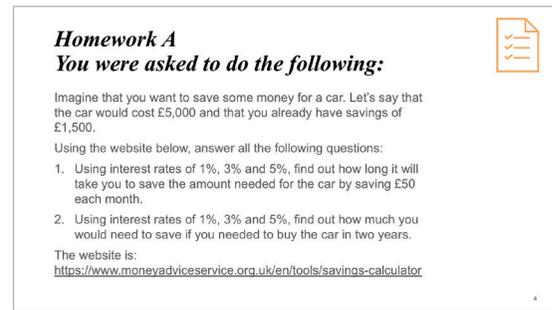
Question 2

- 1% £150
- 2% £145
- 5% £140

REFERENCE SLIDES



Slide 2



Slide 4



Slide 5

INTRODUCE THE CLASS TOPIC

Meet Maya and her family. Maya lives with her parents and her brother, Rohan. Maya is wondering whether it is the right time to open a bank account. She is keen to understand how savings and loans work and has read that interest received and paid is key to understanding both. She wants to know how interest is calculated and how it can affect her. Her great aunt has told her to be aware of credit cards because of the high interest charges. Maya particularly feels that she should understand these, too.

REFERENCE SLIDE



Slide 6

Task 1

CLASS PROBLEM: INTRODUCE TASK 1

Present task 1 for the pupils to answer

EXPLANATORY NOTES

Part of the key here is to ensure that the motives for spending borrowed money have to be first considered. This will answer the question about whether a loan is necessary. Only then will questions about the cost of the loan be considered.



REFERENCE SLIDE

Task 1: loans

Maya knows that she has to be careful in managing her finances. At some point in the future she knows that she may need a loan and also that she may have savings that she would want to place in an account.

Working in pairs, identify five things that should be considered when taking out a loan with a bank. Think what you would do if you wanted to take out a loan for a trip overseas to see a friend, for example.



Slide 7

PUPILS ARE ANSWERING TASK 1





EXPLANATORY NOTES

1. The first question to ask is whether a loan is necessary. Is the trip that important? Other factors should then naturally arise about the ability to repay the loan and finding out the detail of the monthly payments schedule.
2. Shopping around is an important idea and various websites are available to indicate availability and cost of loans.
3. There are other costs typically associated with loans such as arrangement fees. Also, fixed interest loans are available which are generally low interest but these are normally associated with penalty charges if repaid before the end of the agreed period of the loan.
4. Finally, pupils should be encouraged to think about contingency arrangements if they can't afford to repay the loan. It might be worthwhile asking a class question about what they would do. Hopefully, some of the more thoughtful answers would indicate a consideration of the risk associated with the loan and judgements about how to manage it. For example, some pupils may say that they have a secure, part-time job to help repay the loan. It is the idea of a relatively secure income that reflects judgement about one way to mitigate this risk.

REFERENCE SLIDE

Task 1: loans
SOME ANSWERS

1. Do I really need a loan from a bank? Should I be using a bank loan for a trip to see a friend? Is it that important?
2. Should I delay the trip and save up for it instead of taking out a loan?
3. How am I going to pay back the loan? Is it from part-time work or another source? I know the bank will ask this.
4. What is the interest rate charged on the loan? Is it possible to shop around and get the cheapest deal?
5. When am I going to pay back the loan? I know that the longer I have the loan outstanding the greater the total interest I will pay.
6. Can I afford the repayments? I will need to understand the conditions of the loan. There are likely to be fixed payments each month.
7. What are the costs of taking out the loan? Are there other costs involved?
8. What happens if I can't afford the repayments?

Slide 8

INTRODUCTORY TEACHING

This is a review of assumed prior knowledge about growth and decay. The example is of growth and links in to the development of work relating to compound interest. Be sure to explain the expression of percentages as factors to multiply balances; for example, 5% would become a 1.05 factor. This idea is then developed in the next slide using a bank account.

We can use multipliers to work out account balances for any period. So, for 10 years the multiplier would be 1.1^{10}

If the interest rate was 15% instead of 10% then the 10 year multiplier would become 1.15^{10}

REFERENCE SLIDE

Percentages into multipliers

- An interest rate of 10% becomes a multiplier of 1.1 to calculate how an account balance grows after one year.
- We can find out an account balance at the end of two years quickly by using a multiplier of 1.1 raised to the power of 2.

Slide 9

Task 2

CLASS PROBLEM: INTRODUCE TASK 2

An initial explanation of the relationship percentage change and its related multiplier will be needed using the previous slide as an illustration and perhaps this might be elaborated with different examples, using different rates and different compounding periods.

The task relating to the annual interest cost will require pupils to identify the need to use powers or indices to efficiently complete the task. It has been assumed that index operations have been dealt with in earlier studies.

The answer to the third part of the task can be done long hand, using a spreadsheet if it is available. Preferably, it might be better to hint to pupils to use an index to convert the interest rate of 2% to a multiplicative factor of 1.02 and then raising the monthly interest factor of 1.02 to the power 12. It is assumed that they will have used indices before embarking on this exercise.

The following slide explains the use of the compounding formula.

REFERENCE SLIDES

Task 2: bank accounts

Julie was worried that her bank account had shown an overdraft position of £50. She was concerned that she would be charged interest on this amount and had asked you to help her understand what might happen.

The bank statement explained that interest was charged at 2% per month. She asked you to help her understand what she would pay if:

- The overdraft lasted for one month
- The overdraft lasted for two months
- The overdraft lasted for one year

Slide 10

Using interest compounding formulae

The response to part c is most efficiently worked out using the compounding formula:

$$\text{Balance after 12 months} = P \left(\frac{100 + r}{100} \right)^n$$

Where:
P = Principal = initial overdraft balance of £50;
r = interest rate = 2% per month; and
n = period of loan = 12 months.

Slide 11



EXPLANATORY NOTES

Explain the elements of the compounding formula. It is assumed that the use of compounding formulae has been illustrated and discussed before this session. The following notes act as a reminder to this.

The answer to the third part of the task can be done long hand, using a spreadsheet if it is available. Preferably, it might be better to hint to pupils to use an index to convert the interest rate of 2% to a multiplicative factor of 1.02 and then raising the monthly interest factor of 1.02 to the power 12. It is assumed that they will have used indices before embarking on this exercise.

The compounding formula is a short cut for calculating loan or savings balances. Remember that, previously, we calculated account balances year on year by adding interest to them and compounding makes sure that we add interest to the interest. This is important as it is the way that banks operate. Compounding is applied to both savings and loan accounts. With savings accounts, you will receive the interest. With loan accounts, you will pay the interest.

There are three elements to compounding. The Principal (P) is either the initial loan amount that you receive or the initial amount of savings you place on deposit with the bank. The interest, i , is the interest rate received or paid. It is expressed in its usual form, so a 5% rate is inserted in the formula as '5'. The period over which the savings are held on deposit or the loan is held is referred to as 'n'. The way to know how this works is to look at the interest rate since it will say something like '5% per year or 1% per month. If it is a 'per year' interest rate, then 'n' will refer to the number of years the loan is owed or the savings are held on deposit. If it is a 'per month' interest rate then 'n' will refer to the number of months. If it is a 'per day' interest rate (and these do exist!) then 'n' refers to the number of days.

Be sure to emphasise that the compounding formula only applies to loans as long as no repayments are made which isn't usually the case. Similarly, it only applies to savings as long as the initial balance does not change.



PUPILS ARE ANSWERING TASK 2



EXPLANATORY NOTES

If pupils have access to spreadsheets then this is a good but routine exercise for them to do long hand. If they don't have access to a spreadsheet then it will be worthwhile showing the worksheet on a projector to show them just how many calculations are involved and why the mathematical treatment using the formula is so much neater and far shorter!

Emphasise the importance of compounding to ensure that pupils add interest costs on previously incurred interest charges.

REFERENCE SLIDES

Task 2: bank accounts
ANSWER TO A AND B USING THE FORMULA



$$\text{Balance after 1 month} = P \left(\frac{100+r}{100} \right)^n = 50 \left(\frac{100+2}{100} \right)^1 = \text{£}51.00$$

$$\text{Balance after 2 months} = P \left(\frac{100+r}{100} \right)^n = 50 \left(\frac{100+2}{100} \right)^2 = \text{£}51.02$$

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Slide 12

Task 2: bank accounts
ANSWER TO A AND B – ARITHMETIC APPROACH



Overdraft limit	£100
Current overdraft	£50.00
Overdraft interest rate	2% per month
Current overdraft balance	£50.00
Interest charge 2%	£1.00
Balance after 1 month	£51.00
Interest charge 2%	£1.02
Balance after 2 months	£52.02

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Slide 13

Task 2: bank accounts
ANSWER TO C



$$\text{Balance after 1 month} = P \left(\frac{100+r}{100} \right)^n = 50 \left(\frac{100+2}{100} \right)^{12} = \text{£}63.41$$

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Slide 14

TEACHING: THE EFFECT OF COMPOUNDING

The following is an introduction to how the frequency of compounding can affect interest paid or received. The example is with a car loan and so the interest relates to the amount paid.

Please be careful to note that the example is somewhat artificial in that no part repayments are made and the loan is settled in entirety at the end. The reason for the artificiality is that there are no easy formulae to use (and certainly not at this level) that show how interest compounding works with periodic repayments. We will look at an example with a repayment in it later on.

Note: depending on available time, these slides may be used as an additional in-class task.

Let's have a look at how the number of times interest is compounded can affect interest cost on a loan to buy a car. The loan is very straightforward in that we borrow an amount and pay it back in a single instalment one year later. In reality, you would need to make monthly payments on a loan. Our example, though, using just one repayment still illustrates a valid point to make about compounding. Let's see ...

Draw-out the 'apparent', but incorrect, equivalence between the two loans. That is Loan 1 'appears' to be the same as Loan 2 if we multiply Loan 2 by the number of months that the loan would be outstanding (12 months x 1% = 12%). But such arithmetic is incorrect because it ignores the effect of compounding ... Which we see the result of in the next few slides.

Work through the slides.

This point demonstrates that, in effect, what is happening is that interest is charged on interest incurred. With the single repayment loan, there is no interest incurred until the end of the loan and hence no possibility of interest being charged on interest incurred. With the monthly interest loan then interest is incurred from the first month. In the second month the interest charge will be on the outstanding balance at the end of month one which is the loan of £1,000

plus one month's interest of £10 = £1,010. Hence, the balance at the end of the second month is $£1,010 \times 0.001 = £1,020.10$.

REFERENCE SLIDES

Interest costs for any period 

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Slide 15

Interest per period

In the example used to calculate the interest on the overdraft, the interest used was 2% per month.

Interest rates are always based on a period of time. They are based on days, months, or for one year typically.

It is always important to understand what period of time an interest rate is based on because that will affect the total interest cost paid (for a loan) or earned (for savings).

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Slide 16

Example

Joanna is considering getting a loan to pay for a car. She has been quoted for two different types of loan:

- Loan 1: borrow £1,000 at an **annual** interest cost of 12%
- Loan 2: borrow £1,000 at a **monthly** interest cost of 1%

The loan is going to be outstanding for one year and will be repaid in full at the end of the year (there will be no repayments during the year).

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Slide 17

Example

On the face of it, the two loans look equivalent since 1% per month seems to be equal to 12% per year. But they're not. Look at the calculations to see why ...

Loan 1: Interest cost = $12\% \times £1,000 = 0.12 \times £1,000 = £120$

This answer assumes that none of the loan is paid back during the 12 months.

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Slide 18

Example

Loan 2: use the compounding formula:

Loan balance after n periods = $P \left(\frac{100 + r}{100} \right)^n$

Where n is the number of periods of time compounded, in this case 12 months. And so, the loan balance after 12 months if nothing is paid back will be:

$1000 \left(\frac{100 + 1}{100} \right)^{12} = £1,126.83$

The interest cost is then $£1,126.83 - £1,000 = £126.83$

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Slide 19

Example

For comparison we can see that the interest costs are:

- **Loan 1:** £120.00
- **Loan 2:** £126.83

There is a difference of £6.83. This difference will grow:

- the **greater** the amount borrowed
- the **longer** the loan is left outstanding
- the **higher** the interest rate; and
- the **more frequently** the interest is compounded.

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Slide 20

TEACHING OR TASK

This is the final part of the lesson to be taught by the teacher. It prepares pupils for the homework and brings in a more realistic example using a loan with repayments.

The slides use the compounding formula. There is also a spreadsheet version to cross-check the calculations. This is available to you.

Work through the slides.

The major intuitive point to emphasise is that the answer illustrates the point that a repayment, in effect, begins a new loan where we can again apply the compounding formula using the new balance.

The example shows one repayment but this approach can be used for any number of repayments.

REFERENCE SLIDES

Example with a repayment

We can work out what the loan will cost if we have a repayment. Suppose for Loan 2 that a repayment of £500 is made after six months. In this case there are effectively **two loans**: one for the first six months and one for the second six months. Let's see how this works.

Revised loan terms
Loan 2: borrow £1,000 at a monthly interest cost of 1% with two repayments (one after six months of £500 and a final repayment after 12 months that settles the balance).

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Slide 21

Example

Loan 2: finding the balance after six months – use the compounding formula:

$$\text{Balance after 6 months} = P \left(\frac{100 + r}{100} \right)^n = 1000 \left(\frac{100 + 1}{100} \right)^6 = \text{£}1,061.52$$

The interest cost is then $\text{£}1,061.52 - \text{£}1,000 = \text{£}61.52$

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Slide 22

Example

Loan 2: finding the balance after 12 months – use the compounding formula:
A repayment is made of £500. So, at the start of the second period of six months the loan balance will be $\text{£}1,061.52 - \text{£}500 = \text{£}561.52$. And so, the loan balance after the second six-month period will be:

$$\text{Balance after 6 months} = P \left(\frac{100 + r}{100} \right)^n = 561.52 \left(\frac{100 + 1}{100} \right)^6 = \text{£}596.06$$

The interest cost for the second six-month period is then $596.06 - \text{£}561.52 = \text{£}34.54$

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Slide 23

Example summary

Loan balance after six months =	£1,061.52
Loan balance after 12 months with a repayment of £500 is	£596.06
Interest paid for first six months =	£61.52
Interest paid for second six months =	£34.54
Total interest paid	£96.06

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Slide 24

Homework for lesson 2

We've now got a fairly good understanding of how to use compounding to understand interest and loan repayments. Have a go at the homework which we'll deal with in the next lesson.

Homework B: Ask the pupils to complete the homework. This is included in the Tasks folder to print off if necessary.

PART 1

Joanna has heard about payday loans where interest rates can become very high, as much as 40% per month. Joanna is not considering taking out a payday loan but was wondering how mathematics could help her understand the implications of taking out one of the following loans.

Loan 1: borrow £200 at a monthly interest cost of 40%. Loan 1 is taken out on 1 July and will be repaid over a two-month period. The first repayment instalment due on 31 July will repay half of the original loan: £100, plus outstanding interest at that time. The second instalment will settle the outstanding loan and interest balance on 31 August.

1. What is the amount of the second instalment payment?
2. What is the total interest cost in £ of the loan?

PART 2

Joanna is considering a different loan.

Loan 2: borrow £200 at a daily interest cost of 1.5%. Loan 2 is taken out on 1 July and will be repaid over a two-week period. The first repayment instalment due on 8 July will repay half of the original loan: £100, plus outstanding interest at that time. The second instalment will settle the outstanding loan and interest balance on 15 July.

1. What is the amount of the final instalment payment?
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REFERENCE SLIDES

**Homework B
Part 1**

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Slide 25

**Homework B
Part 2**

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Slide 26

Before lesson 2

HOMEWORK B FOR THE PUPILS: THE FOLLOWING WORK TO BE SET BEFORE THE LESSON:**PART 1**

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Encourage pupils to use both the compound formula approach and to confirm their answers on a spreadsheet if available.

REFERENCE SLIDES

**Homework B
Part 1**

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1. What is the amount of the second instalment payment?
2. What is the total interest cost in £ of the loan?

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**Homework B
Part 2**

Joanna is considering a different loan.

Loan 2: borrow £200 at a daily interest cost of 1.5%. Loan 2 is taken out on 1 July and will be repaid over a two-week period. The first repayment instalment due on 8 July will repay half of the original loan: £100 plus outstanding interest at that time. The second instalment will settle the outstanding loan and interest balance on 15 July.

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Slide 29

Lesson time

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Teaching and activities

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Ask for volunteers to say or present their homework results to the class. (How this is tackled by the teacher will vary depending on how willing the class is to present their ideas). It may be preferable to have group ideas presented or to choose one or two pieces of homework for the teacher to present.



HOMEWORK

PART 1

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PART 2

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Encourage pupils to use both the compound formula approach and to confirm their answers on a spreadsheet if available.



EXPLANATORY NOTE

Work through the slides.

Although we should not advocate payday loans it is important to understand them. One reason a person may take out a payday loan is that, in the face of not having enough money to pay bills for example, the total interest cost of £35.37 for a loan of £200 might be preferable than not being able to afford food. The reality and key problem of the choice for individuals choosing payday loans is one of liquidity (not having available cash) which for them takes priority over interest costs. The interest rate may be high but if the loan is repaid quickly then the interest cost in terms of £ is arguably not large when seen as a charge for the benefit to gain liquidity. This is not advocacy for payday loans but an explanation of why they might be used.

Payday loans are part of the wide spectrum of loans that are available. The general rule is that the higher the risk of the individual not meeting the commitments under the loan, the higher the interest rate they will pay. Risk is judged on a number of factors: previous credit history, security for the loan (eg, a car or house), financial capacity, and so on. Money lenders must be licensed by the Financial Conduct Authority to offer credit: <https://www.gov.uk/offering-credit-consumers-law>

REFERENCE SLIDES

Homework B Part 1

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Slide 28

Homework B Part 2

Joanna is considering a different loan.

Loan 2: borrow £200 at a daily interest cost of 1.5%. Loan 2 is taken out on 1 July and will be repaid over a two-week period. The first repayment instalment due on 8 July will repay half of the original loan: £100 plus outstanding interest at that time. The second instalment will settle the outstanding loan and interest balance on 15 July.

1. What is the amount of the final instalment payment?
2. What is the total interest cost in £ of the loan?

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Homework B Class discussion

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Homework B Answer, part 1

Using the formula approach, here is the loan balance after one month

$$\text{Balance after 1 month} = P \left(\frac{100+r}{100} \right)^n = 200 \left(\frac{100+40}{100} \right)^1 = £280$$

A repayment of £100 is made to bring the loan balance down to:
 $£280 - £100 = £180$.

This becomes the new balance to begin the second period of the loan
 Interest costs for the first 1 month period are therefore $£280 - £200 = £80$

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Homework B Answer, part 1

Using the compounding formula, we see that the loan balance after the second one month period is:

$$\text{Balance after 2 months} = P \left(\frac{100+r}{100} \right)^n = 180 \left(\frac{100+40}{100} \right)^1 = £252$$

Interest costs for the second one month period is therefore $£252 - £180 = £72$
 The answers to the questions in the homework are therefore:

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Homework B Answer

First instalment	£100.00
Final instalment	£252.00: answer to question 1
Total interest paid	£152.00: answer to question 2

Total interest costs are $80 + 72 = £152$
 Total amount repaid for a loan of £200 is £352

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Homework B Answer, part 2

Using the formula approach, here is the loan balance after one week

$$\text{Balance after 1 week} = P \left(\frac{100+r}{100} \right)^n = 200 \left(\frac{100+1.5}{100} \right)^7 = £221.97$$

A repayment of £100 is made to bring the loan balance down to:
 $£221.97 - £100 = £121.97$
 This becomes the new balance to begin the second period of the loan.
 Interest costs for the first week are therefore $£221.97 - £200 = £21.97$

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Homework B Answer, part 2

Using the compounding formula, we see that the loan balance after the final 7 day period is:

$$\text{Balance after 2 weeks} = P \left(\frac{100+r}{100} \right)^n = 121.97 \left(\frac{100+1.5}{100} \right)^7 = £135.37$$

Interest costs for the second week are therefore $£135.37 - £121.97 = £13.40$
 The answers to the questions in the homework are therefore:

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REFERENCE SLIDES (CONT)

Homework B Answer	
First instalment	£100.00
Final instalment	£135.37: answer to question 1
Total interest paid	£35.37: answer to question 2
Total interest costs are therefore $21.97 + 13.40 = £35.37$	
Total amount repaid for loan of £200 is £235.37	

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RECAP ON WHAT HAS ALREADY BEEN DONE

IF NEEDED, RECAP THIS FROM THE PREVIOUS SESSION: *Meet Maya and her family. Maya lives with her parents and her brother, Rohan. Maya is wondering whether it is the right time to open a bank account. She is keen to understand how savings and loans work and has read that interest received and paid is key to understanding both. She wants to know how interest is calculated and how it can affect her. Her great aunt has told her to be aware of credit cards because of the high interest charges. Maya feels that she should understand these too.*

REFERENCE SLIDE



Slide 6

INTRODUCE AND MOTIVATE THE NEXT TASK

We now turn our attention to savings accounts to illustrate that compounding also works here. It will be important in what follows to pay attention to the detail of what is on offer. Only by looking at the detail can we fully understand how to make the right decisions to choose between alternatives. Our mathematical analysis will help us in this respect.



EXPLANATORY NOTES

The idea behind the problems is to get the pupils to be very comfortable about thinking about the three elements of compounding (principal, interest and period), how these can vary, and what effect variations in one element has on the remaining two. These examples reflect the level of some of the questions used in examinations.

If running short of time then omit part b.

REFERENCE SLIDES

Task 3: savings accounts 

Reena is in a different position to Joanna and has just been given a gift of £2,000 from her gran to help her with future university costs. She plans to go to university in three years. She has investigated two savings accounts that she is considering depositing her £2,000 into.

Details of the two accounts are on the next slide.

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Task 3: savings accounts 

Savings account 1

- Invest up to £3,000. Must be left in the account for three years. Annual interest paid is 2% and tax of 20% is deducted from the interest earned.

Savings account 2

- Exclusive to savers under 18. Invest any amount. Withdrawals at any time, tax free. Interest rate paid is 0.85% every six months.

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Task 3: savings accounts 

Work in pairs. Reena wants to achieve a balance in her account of £2,100 before she withdraws her savings in three years' time. Using the savings account details provided, answer the following questions:

- What balance will be in Savings account 1 after three years?
- What initial savings must Reena deposit in Savings account 1 to achieve a balance of £2,100 in three years?
- If Reena can only afford to deposit £2,000, what must the interest rate rise to in Savings account 1 to achieve a balance of £2,100 after three years?
- If Reena can only afford to deposit £2,000 and wants to use Savings account 2, how long must she invest for to get a balance of £2,100?

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Task 3

PUPILS ARE ANSWERING TASK 3



The answer to the problem is ... (go through the slides)

First, here are the answers to the calculations.

EXPLANATORY NOTE PART A



It is best to do the long-hand calculation to demonstrate to pupils how to adjust the interest rate for tax deductions which will then enable them to use the compounding formula directly.

REFERENCE SLIDES

Task 3: savings accounts
ANSWER, A

What balance will be on Savings account 1 after three years?

Savings account 1

Annual interest rate	2.00%
Amount invested	£2,000.00
Tax rate on interest	20.00% (Paid when interest paid)

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Task 3: savings accounts
ANSWER, A

	£	
First year	2,000.00	
Interest	40.00	
Tax	-8.00	
Balance after 1 year	2,032.00	
Second year	2,032.00	
Interest	40.64	
Tax	-8.13	
Balance after 2 years	2,064.51	
Third year	2,064.51	
Interest	41.29	
Tax	-8.26	
Balance after 3 years	2,097.54	

The first period tax is calculated as:
20% x £40 or 0.2 x £40 = £8

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Task 3: savings accounts
ANSWER, A

Solving the Savings account 1 problem using the compounding formula

The tax rate may be deducted directly from the interest rate to derive an 'after tax net interest rate' as follows:
 $r = 2\% - 0.2 \times 2\% = 1.6\%$. This can then be used in the compounding formula directly as:

$$\text{Balance after 3 years} = 2,000 \left(\frac{100 + 1.6}{100} \right)^3 = £2,097.54$$

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The answer to the problem is ... (go through the slides)

First, here are the answers to the calculations.

EXPLANATORY NOTE PART B

This can be calculated from the compounding formula but requires the use of indices manipulation which will be beyond the group being taught. It is answered by trial and error or trial and improvement and the purpose is to get pupils to practise how the basic relationships work in the compounding formula. Pupils will need access to a calculator that is capable of answering $y=x^n$ problems.

The teacher might illustrate how the answer can be arrived at by using the accompanying spreadsheet.

REFERENCE SLIDES

Task 3: savings accounts
ANSWER, B

What initial savings must Reena deposit in Savings account 1 to achieve a balance of £2,100 in three years?

Savings account 1

Annual interest rate	2.00%
Amount invested	?????
Tax rate on interest	20.00% (Paid when interest paid)

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Task 3: savings accounts
ANSWER, B

	£	
First year	2,003.00	
Interest	40.06	
Tax	-8.01	
Balance after 1 year	2,035.05	
Second year	2,035.05	
Interest	40.70	
Tax	-8.14	
Balance after 2 years	2,067.61	
Third year	2,067.61	
Interest	41.35	
Tax	-8.27	
Balance after 3 years	2,100.69	

The answer is £2,003 and can be found by trial and error or trial and improvement

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Slide 45

The answer to the problem is ... (go through the slides)

First, here are the answers to the calculations.

EXPLANATORY NOTE PART C

Again, this is another trial and error approach to solve the problem and get pupils used to understanding the relationships between interest rates, amounts owed/invested, and time.



REFERENCE SLIDES

Task 3: savings accounts
ANSWER, C

If Reena can only afford to deposit £2,000, what must the interest rate rise to in Savings account 1 to achieve a balance of £2,100 after three years?

Savings account 1

Annual interest rate	?????
Amount invested	£2,000.00
Tax rate on interest	20.00% (Paid when interest paid)

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Task 3: savings accounts
ANSWER, C

	£	
First year	2,000.00	
Interest	41.00	
Tax	-8.20	
Balance after 1 year	2,032.80	
Second year	2,033.80	
Interest	44.67	
Tax	-8.33	
Balance after 2 years	2,066.14	
Third year	2,066.14	
Interest	42.36	
Tax	-8.47	
Balance after 3 years	2,100.02	

This answer uses an interest rate of 2.05%

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Slide 47

The answer to the problem is ... (go through the slides)

First, here are the answers to the calculations.



EXPLANATORY NOTE PART D

The answer to this is in two parts. Pupils need to establish what the year three balance will be investing £2,100 to discover whether they should then lengthen or shorten the time period over which the investment is made. Using the initial data in the question, the balance after three years is £2,104.19 and hence the period to achieve £2,100 is less than three years and so the value of n must be reduced.

Note the use of the compounding formula: the period over which the interest is calculated must relate to the period value for 'n'. That is, 'n' is represented as '6' or 6 x 1/2 years. It would be a mistake to write this as '3' to represent three years.

This is a potentially tricky calculation because pupils will need to realise that the value of n in the compounding formula will now involve a decimal point. In the initial data n=6. It must be lowered to reflect a shorter period, the correct answer is to use n = 5.75 (equal to five years and nine months). The instructor may need to give some hints to focus attention on how the value of n may be changed to provide the answer required.

Pupils, using trial and error in the compounding formula, would have worked out - had they tried a value of n greater than 6 - that a value lower than 6 needed to be tried. This is a useful exercise for pupils to practise the relationships in the compounding formula to understand how bank accounts paying interest work.

Of course, the calculations work for loans. However, stress to pupils that there are further complications in loan accounts that need to be taken account of, such as periodic repayments, which mean that straightforward calculations of the type used here need adjusting to account for such complicating factors.

REFERENCE SLIDES

Task 3: savings accounts
ANSWER, D

If Reena can only afford to deposit £2,000 and wants to use Savings account 2, how long must she invest for to get a balance of £2,100?

We first need to calculate the account balance after three years using the existing data.

Interest rate	0.85% every 6 months
Amount invested	£2,000.00
Tax rate on interest	nil

Use the compounding formula:

$$£2,000 \left(\frac{100 + 0.85}{100} \right)^6 = £2,104.10$$

The period of investment has to reduce to produce a balance of £2,100.

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Task 3: savings accounts
ANSWER, D

The second stage of the calculation is to find, by trial and error or trial and improvement, what value of n is needed in the compounding formula to produce an account balance of £2,100. The next value of n chosen must be lower than six.

Interest rate	0.85% Every 6 months
Amount invested	£2,000.00
Tax rate on interest	nil

Use the compounding formula:

$$£2,000 \left(\frac{100 + 0.85}{100} \right)^{5.75} = £2,099.74$$

The answer is £2,099.74 which is a close enough approximation to £2,100.

The period of 5.75 years is equivalent to five years and nine months.

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SOME FINAL THOUGHTS

Debit cards withdraw money immediately from your bank account (if you have enough money in the bank account). They reduce your bank balance almost immediately and are similar in that respect to writing a cheque, withdrawing cash from a machine (automated teller) or paying contactless by phone.

Credit cards are short-term loans that require minimum payments to be made each month. Outstanding balances are charged interest, which is normally quite high (at about 1.5%-2% per month, or even higher). Credit cards are best used when the full balance is paid off each month. When that is done interest is not charged. It is unwise to use credit cards and maintain a loan balance over a long period of time.

The following website contains good details on how interest is calculated

<https://www.hsbc.co.uk/1/2/credit-cards/credit-card/interest-rates>

REFERENCE SLIDES

Credit and debit cards

Credit and debit cards are ways to pay for things without using cash but they are not identical.

- Debit cards are a way to make immediate payments from your bank account.
- Credit cards are mostly used to make payments for things but are in effect short-term loans and do not withdraw cash immediately from your bank account.

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Some interesting facts about credit cards

Credit cards are a convenient way to manage finances. According to Creditcards.com:

- During March 2016 alone, £12.6bn was spent on credit cards
- There are 60m cards in issue in the UK
- Each household, on average, owed £2,469 on credit cards in November 2016. Many will pay this off without incurring interest, but ...
- It would take 25 years and 6 months to repay the average debt if only the minimum repayment each month was made!

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Resources

1. LESSON PLAN (THIS DOCUMENT)
2. POWERPOINT
3. TASK HANDOUTS
4. SPREADSHEET

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