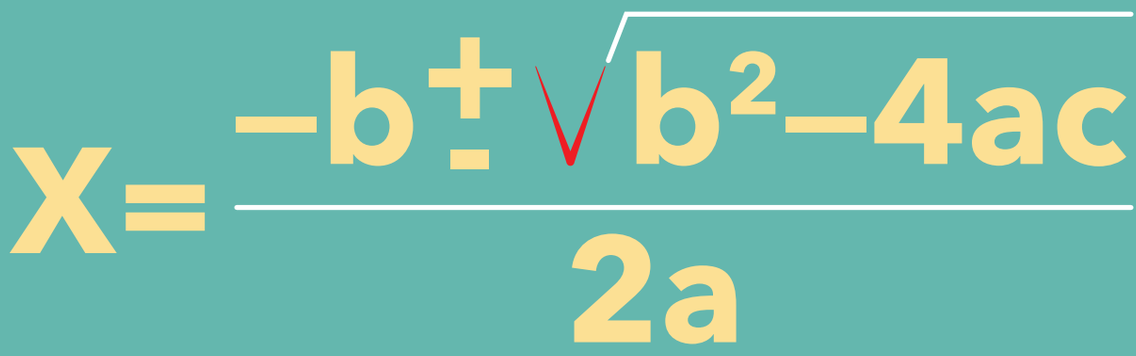
ICAEW
LESSON PLANS



Algebra

MATHEMATICS KEY STAGE 4

icaew.com/ICAEWmaths

Introduction

SUMMARY:

Use algebra to help understand how to run a business.

STRUCTURE:

The content is divided into two (approximately) 60 minute lessons.

PRESENTATION:

Italicised text are suggested scripts for the teacher to say. There are explanatory notes to aid quick understanding of some of the finance material.

GCSE ASSESSMENT OBJECTIVES ADDRESSED IN THE LESSON[1]

The mathematical content specifications in this presentation are those used in the Mathematics GCSE Subject content and assessment objectives and are identified in red.

A2: substitute numerical values into formulae and expressions, including scientific formulae

A3: understand and use the concepts and vocabulary of expressions, equations, formulae, identities inequalities, terms and factors

A4: simplify and manipulate algebraic expressions (including those involving surds and algebraic fractions) by: collecting like terms

A5: understand and use standard mathematical formulae; rearrange formulae to change the subject

A23: generate terms of a sequence from either a term-to-term or a position-to-term rule

A24: recognise and use sequences of triangular, square and cube numbers, simple arithmetic progressions, Fibonacci type sequences, quadratic sequences, and simple geometric progressions (r n where n is an integer, and r is a rational number > 0 or a surd) and other sequences

A25: deduce expressions to calculate the nth term of linear and quadratic sequences.

[1] From government specification content where:

- All students will develop confidence and competence with the content identified by standard type
- All students will be assessed on the content identified by the standard and the underlined type; more highly attaining students will develop confidence and competence with all of this content
- Only the more highly attaining students will be assessed on the content identified by **bold** type. The highest attaining students will develop confidence and competence with the **bold** content.

Before lesson 1

HOMEWORK FOR THE PUPILS

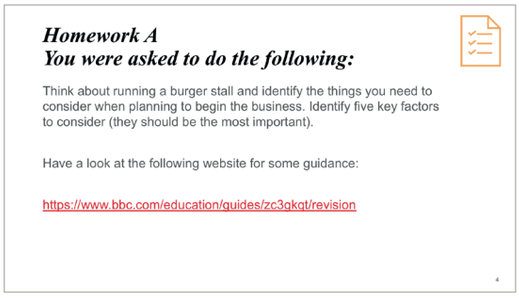
Think about running a burger stall and identify the things you need to consider when planning to begin the business. Identify five key factors to consider (they should be the most important).

Have a look at the following website for some guidance:
https://www.bbc.com/education/guides/zc3gkqt/revision

FOR THE TEACHER

You can offer guidance to pupils by encouraging them to use a search engine and search for 'starting up a business'. There are many sites available with enough information to provide some basic ideas. The key issue is to encourage them to think about all of the key aspects of a business that will make it successful and then for them to begin to think about which might be the most important.

REFERENCE SLIDE



Slide 4

Lesson time

GENERAL NOTES

The associated powerpoint presentation has supporting script notes to help you. These can best be viewed by clicking 'View/Notes Page' in powerpoint. Items in *italics* are a proposed script for you to say. The content of the lesson plan follows closely the notes contained in the powerpoint.

Teaching and activities

CLASS DISCUSSION: INTRODUCE DISCUSSION ON THE HOMEWORK

Ask for volunteers to say or present their homework results to the class. (How this is tackled by the teacher will vary depending on how willing the class is to present their ideas). It may be preferable to have group ideas presented or to choose one or two pieces of homework for the teacher to present.

EXPLANATORY NOTES

A key skill being developed is in using financial websites confidently and in discovering that there is useful information and web content available.

HOMEWORK FOR THE PUPILS

The following work to be set before the lesson:

Think about running a burger stall and identify the things you need to consider when planning to begin the business. Identify five key factors to consider (they should be the most important).

Have a look at the following website for some guidance:
https://www.bbc.com/education/guides/zc3gkqt/revision

HERE ARE SOME POSSIBLE ANSWERS:

1. What location should I use for the stall?
2. Customers: how many will I need? What sales will I achieve?
3. Will I need staff? Would they be part time or full time?
4. What money will I have to invest? The business needs to make a profit to survive and may need money to start it (buying stock, for instance, or paying rent).
5. Will I need a plan for the business? A plan will show what you anticipate happening over the next 12 months, including budgeting to finance the business and to make sure it generates enough cash to pay the bills.

Notes to the answers:

1. Location will be important as you will need to ensure you have enough customers passing by.
2. You will need to estimate how many customers or what level of sales are needed in order to earn a living.

3. If the business becomes big or has busy periods then extra staff will be needed.
4. All businesses take a while to earn profit. Cash will be needed at the outset to invest, pay staff, and so on.
5. A plan is essential month by month to make sure that the business has realistic targets to aim for.

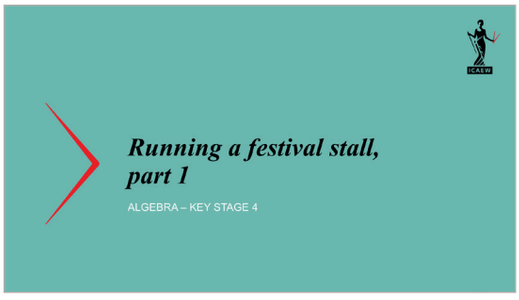
POINTS FOR THE TEACHER:

Pupils will raise a wide variety of points and these should be encouraged since they help develop skills of offering a wide perspective from which the teacher can help them choose the key items. The key items are likely to be those identified in the slide.

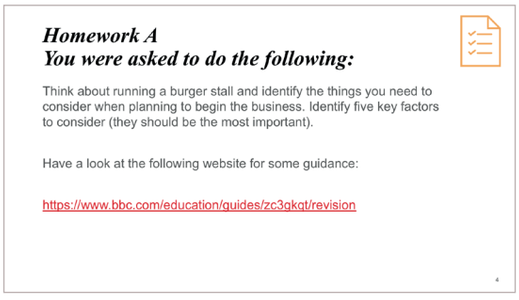
Some non-financial aspects might be important, also, such as:

1. Developing a lifestyle business (running a business that is also your hobby) that cannot be done working for a company
2. Being your own boss and in charge of your own destiny
3. Being flexible about working
4. Leaving the rat race of corporate culture

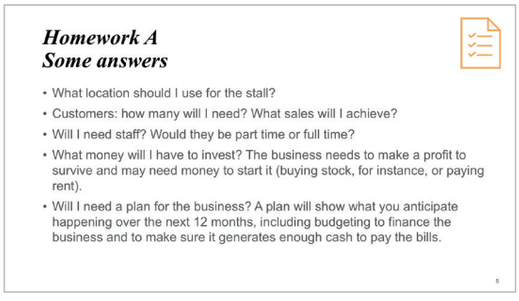
REFERENCE SLIDES



Slide 2



Slide 4



Slide 5

INTRODUCE THE CLASS TOPIC

Let's explore the use of algebra to see how we can provide some help to Janette who is running a burger stall. We are going to look in detail about what is needed to run a business.

Janette runs a burger stall in her local town. She sets up in the market square every day at 10:00 and closes about six hours later, or when she feels that it is likely that there will be no further custom.

There are a lot of activities in running a stall. In many ways, it has many of the features of running a business of any size and we can see what that entails.

REFERENCE SLIDE

Slide 6

Task 1

CLASS PROBLEM: INTRODUCE TASK 1

One of Janette's major problems is to know how many burgers to buy each day. Generally, because she uses a freezer to store her stock, she tends to buy enough burgers to last three or four days. Janette knows that she can get a discount if she buys a lot of burgers and this suits her because she does not have to go and collect them so frequently.

***Working in pairs**, what factors do you think she should consider to decide how much stock of burgers to buy?*

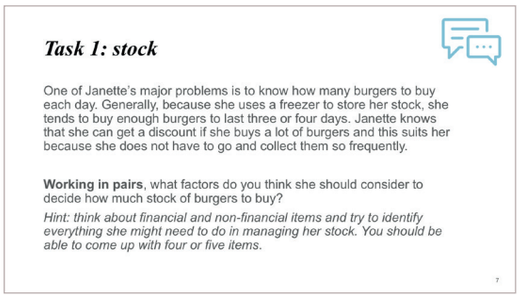
Hint: think about financial and non-financial items and try to identify everything she might need to do in managing her stock. You should be able to come up with four or five items.

EXPLANATORY NOTES

The key aim here is to get the pupils to identify as many issues as possible. Identification of the boundary of the problem is an important first task before analysis of the issues begins. This is a straightforward task that can be as detailed as possible and it is important that pupils are encouraged to go into as much detail as they like.

Say the following: *Try and identify as many tasks as possible. Be as detailed as you can and break down the tasks into smaller and smaller tasks to reach a point where nothing is missed out.*

REFERENCE SLIDE



Slide 7

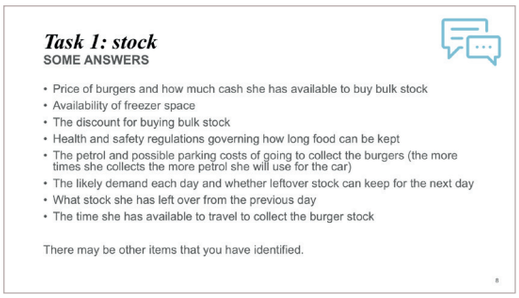
PUPILS ARE ANSWERING TASK 1

EXPLANATORY NOTES

The point to bear in mind is that there is a lot of detailed activity for Janette to run her business each day. The more detail the pupils are encouraged to think about, the better. For both maths and finance, attention to detail is critical in resolving problems.

Whilst not listed, pupils might mention the fact that the significant amount of time that Janette takes to run her business means that she has less time for leisure activities. This is an important issue but it is not one we will otherwise highlight.

REFERENCE SLIDE



Slide 8

INTRODUCTORY TEACHING

The scene involves Janette going to a festival to pitch her stall. This will involve a lot of planning and algebra will help significantly in this respect.

EXPLANATORY NOTES

This, and the next few slides, are instructional in terms of setting up the context in which algebra skills are going to be exercised. The context is straightforward and a key point will be to emphasise to the pupils that running a business is what many of them will end up doing and that the profits they earn will be their 'wages'.

For people running small businesses, profit is their 'wages' or 'earnings'. Profit is what is left over after paying all of the costs of running a business. As with running her stall at the normal pitch in the town centre, it is important to gather all the facts to make a judgement about whether it is worthwhile to attend the festival. Again, attention to detail is going to be critical. We're going to use some maths, using this detail, to help Janette make some important decisions while she is at the festival.

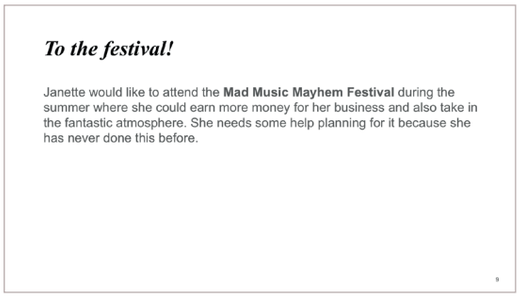
Slide 11 is essentially a reminder of the results from task 1. The important additional point is the distinction between costs that vary with the level of sales (variable costs) and those that do not vary with sales. This distinction will be important in building up to the mathematical analysis of the problem.

Slide 12 is an important slide as the distinction between fixed and variable costs will indicate to Janette what factors will influence her costs and therefore profits. This will be important in understanding how to create an algebraic expression for profit to be done in a later slide.

It is important that Janette makes a profit because these are her wages or earnings. Profit is defined as the surplus of money received from sales less the costs.

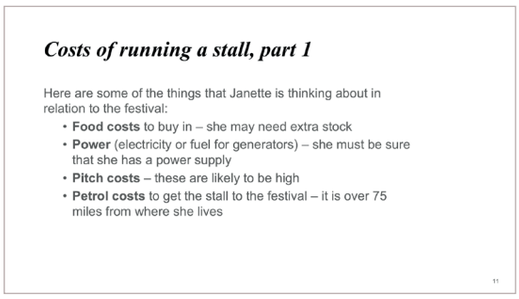
Make a point to say that this is a simplified example because there may be other costs involved. The other important point to make is that we need to help Janette to understand how her profit changes with different circumstances, like going to the festival. This is where algebra is going to help, as we will see.

REFERENCE SLIDES

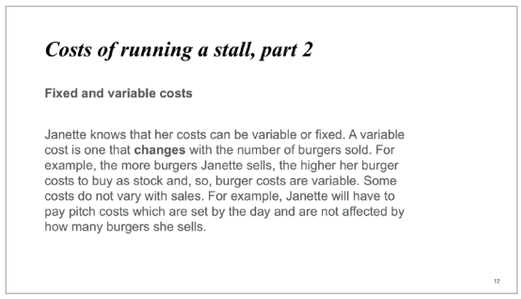


Slide 9

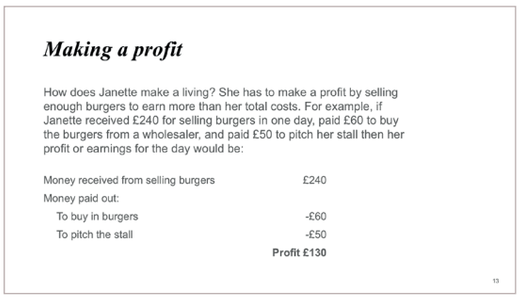
Slide 10



Slide 11



Slide 12



Slide 13

SETTING UP THE CONTEXT FOR THE NEXT TASK: CREATING A FORMULA FOR PROFIT

We've simplified the problem and, so, we do not directly consider other food costs associated with the burger such as bread buns or sauces.

Indicate the relationships involved in diagram 1.

In diagram 1, the value of burger sales increases with the number of burgers sold, as does the amount of variable cost. Sales = price per burger x number of burgers sold.

Variable costs are the costs of the burgers that Janette buys for resale. Variable cost = cost per burger x number of burgers sold. The fixed costs, on the other hand, do not change with the number of burgers sold. Fixed costs at the festival are the pitch costs. Fixed costs = pitch costs per day x number of days using the pitch.

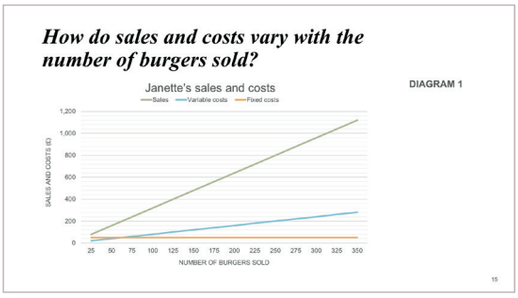
Indicate the relationships involved in diagram 2.

In diagram 2, the profit line is plotted. Profit is calculated as sales less variable costs less fixed costs. It is important to realise in this diagram that the profit (orange) line is the vertical difference between sales (blue line) and total costs (green line). Specifically, this can be demonstrated where zero profit arises when sales equals total costs.

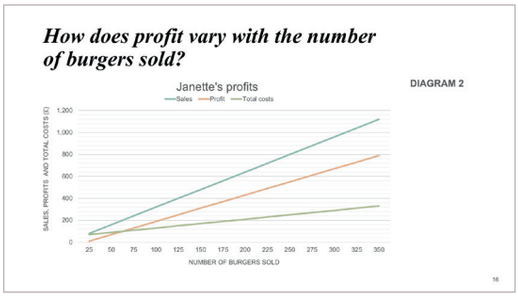
Say the following: *Underlying these diagrams are relationships between sales, costs and profit that we can represent mathematically. Now that we understand the profit relationships in Janette's business, we can begin to help her with some decisions.*

REFERENCE SLIDES

Slide 14



Slide 15



Slide 16

Task 2

CLASS PROBLEM: INTRODUCE TASK 2

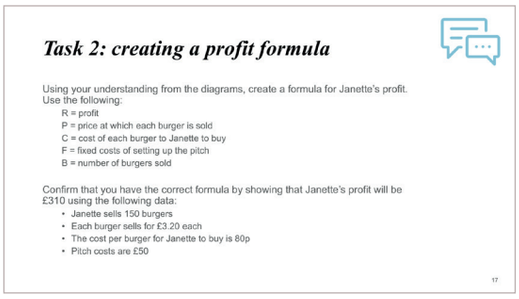
This reinforces what has been covered in the previous slide.

It might be worthwhile hinting that the revenue (sales) that Janette receives will depend on the number of burgers sold. The variable costs will also depend on the number of burgers sold but the fixed costs will not alter. This should give pupils an idea about how to create the formula for profit. If they are struggling with this, refer them to slide 9 which shows the basic

profit relationship. They then have to use this basic relationship to create a formula to calculate profit for any level of burger sales, B.

Note on fixed costs: these do not vary with sales. They are sometimes referred to as 'period costs' since they depend – like rent or pitch costs – on an amount of time. To be accurate, they can vary with time but they are fixed in relation to sales.

REFERENCE SLIDE



Slide 17

PUPILS ARE ANSWERING TASK 2

TASK 2 ANSWER AND FURTHER EXAMPLE

This formula recognises that profit depends on the number of burgers sold, B. Pupils may have developed a variety of different formats for this formula.

It might be worthwhile pointing out that the part of the formula, (P-C), is given a special term: contribution.

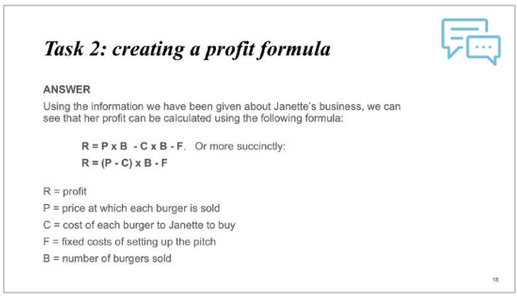
The advantage of using the formula is that it allows us to calculate any profit figure for any price, burger cost, and fixed cost. The formula is general and can be applied to any situation. That is the great power of formulae!

Reinforcement: *Let's have a look at another example with a different level of burger sales.*

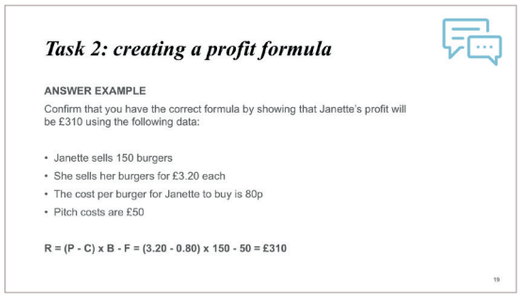
Note that fixed costs do not change.

Clearly, the more burgers that Janette can sell the more profit she can make.

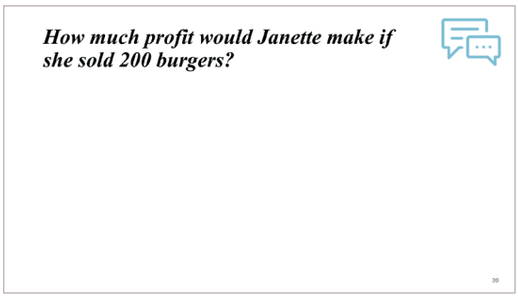
REFERENCE SLIDES



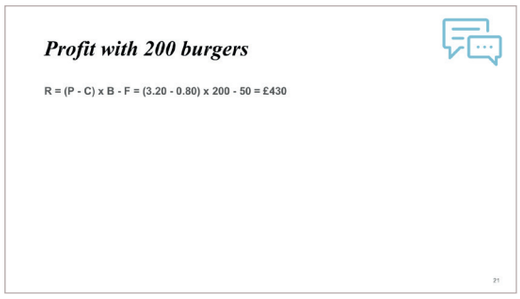
Slide 18



Slide 19



Slide 20



Slide 21

Task 3

CLASS PROBLEM: INTRODUCE TASK 3

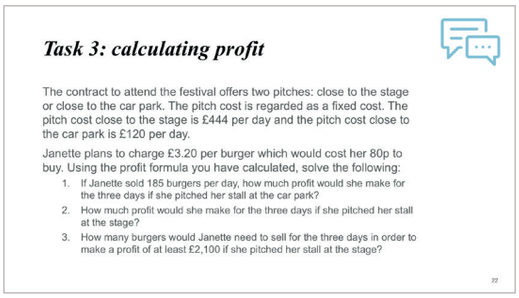
Now, let's explore how we can help Janette with her decisions about the festival. We'll use the profit formula that we've calculated to answer a few questions.

Work in your pairs together on these problems.

EXPLANATORY NOTES

Note that questions 1 and 2 should be fairly routine and are designed to build up some confidence. Question 3 will require pupils to make B the subject of the formula and will therefore require some skills of rearranging a formula in order to complete it successfully. It might be worthwhile to explain that B should be made the subject of the formula as a hint to the pupils.

REFERENCE SLIDE



Slide 22

PUPILS ARE ANSWERING TASK 3

TASK 3 ANSWER AND FURTHER EXAMPLE

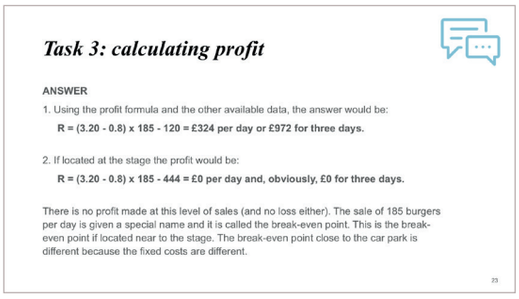
The break-even point is an important concept to understand. Remember that profits are equivalent to earnings for Janette and she needs to know just how many burgers to sell before she earns any money for herself. Break-even recognises that profits for Janette are *only calculated after all of her other costs are paid*. Only by knowing the break-even point can Janette be clear about how many burgers to buy, how much trade or sales to anticipate or work toward, and how both of these need to be considered to ensure that the difference she makes between buying and selling burgers is enough to pay for her pitch costs.

This final point is dealt with further in the answer to question 3.

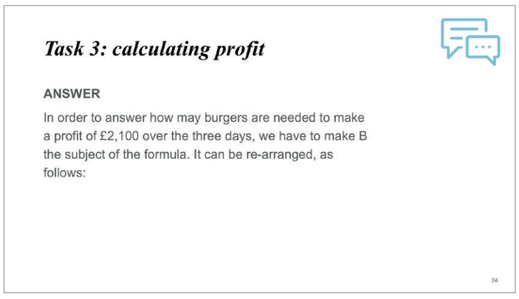
This slide can be used as a hint if pupils are not certain about how to proceed.

We need then to find values for R, F, P and C in order to determine B. If Janette wants a profit of £2,100 then that figure becomes the value of R. The values of C, P, and F are already given. The values can then be inserted into the derived formula. The resulting value for B is the number of burgers that have to be sold if Janette wishes to earn a profit of £2,100.

REFERENCE SLIDES

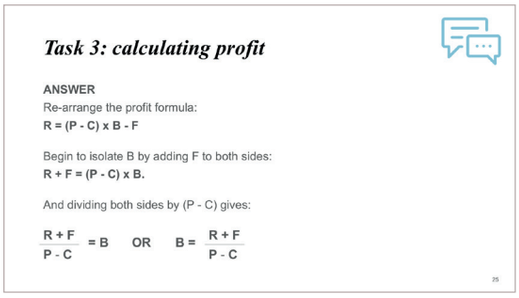


Slide 23

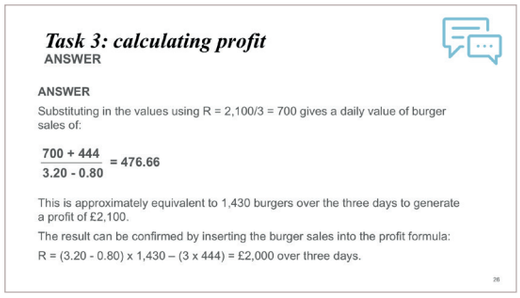


Slide 24

REFERENCE SLIDES (CONT)



Slide 25



Slide 26

Homework for lesson 2

We've now got a fairly good understanding of how profits are earned. Let's have a look at how Janette can make the best use of her time at the festival.

Homework: *It will be vital for Janette to make sure that her visit to the festival will be profitable. She cannot afford to incur a loss! She wants to avoid making the mistakes that will give rise to a loss and one way of doing this is to understand what she needs to do to make sure her customers are happy.*

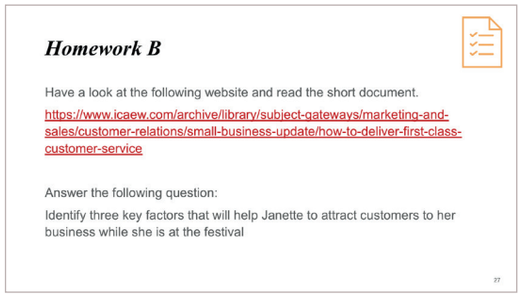
Have a look at the following website and read the short document.

https://www.icaew.com/archive/library/subject-gateways/marketing-and-sales/customer-relations/small-business-update/how-to-deliver-first-class-customer-service

Answer the following question:

Identify three key factors that will help Janette to attract customers to her business while she is at the festival

REFERENCE SLIDE



Slide 27

Lesson 2

General notes: The associated powerpoint presentation has supporting script notes to help you. These can best be viewed by clicking 'View/Notes Page' in powerpoint. Items in italics are a proposed script for you to say. The content of the lesson plan follows closely the notes contained in the powerpoint.

Teaching activities

CLASS DISCUSSION: INTRODUCE DISCUSSION ON THE HOMEWORK

Ask for volunteers to say or present their homework results to the class. (How this is tackled by the teacher will vary depending on how willing the class is to present their ideas). It may be preferable to have group ideas presented or to choose one or two pieces of homework for the teacher to present.

Homework: *It will be vital for Janette to make sure that her visit to the festival will be profitable. She cannot afford to incur a loss! She wants to avoid making the mistakes that will give rise to a loss and one way of doing this is to understand what she needs to do to make sure her customers are happy.*

Have a look at the following website and read the short document.

https://www.icaew.com/archive/library/subject-gateways/marketing-and-sales/customer-relations/small-business-update/how-to-deliver-first-class-customer-service

Answer the following question:

Identify three key factors that will help Janette to attract customers to her business whilst she is at the festival

EXPLANATORY NOTE

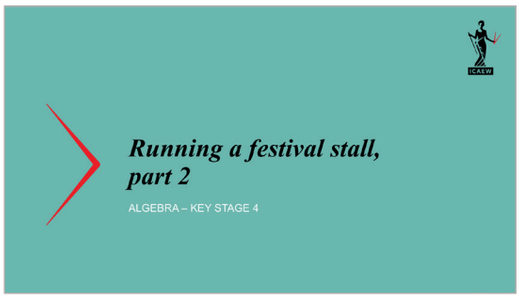
The following key factors might be relevant:

1. Be better than the competition
2. Understanding your customers
3. Offer excellent customer service

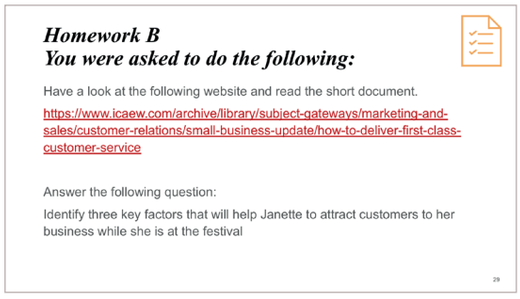
Further explanation

1. There will be a lot of food stalls at the festival and Janette must make her stall inviting. This might involve ensuring that it looks good, the food smells good and that her prices are not too high compared with what others are offering.
2. Janette must understand what customers want. She won't know people at a personal level at the festival but she will have an understanding of what it takes to sell burgers. This might involve ensuring she uses the best products, has appropriate condiments (tomato and other sauces, and sells other products that might be needed such as soft drinks.
3. Janette must offer good service standards. She must have enough staff to service customers. She must have enough food cooking to make sure that they do not wait too long. She must have enough stock of everything she sells to make sure that she doesn't disappoint customers.

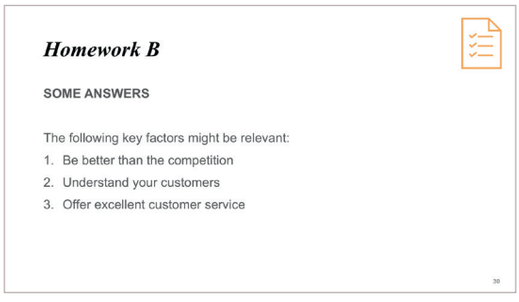
REFERENCE SLIDES



Slide 28



Slide 29



Slide 30

INTRODUCE AND MOTIVATE THE NEXT TASK

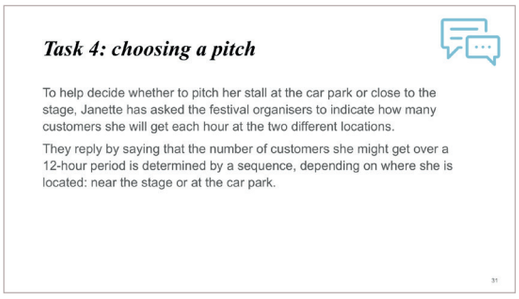
This slide introduces the use of sequences into the problem. Sequences are used to estimate the demand for burgers and the sequences themselves represent the number of customers arriving at the burger stall for each hour. It is important to stress that the sequences are numbers of customers for successive hours since we will ask pupils to answer problems in these terms.

The quadratic sequence problem is clearly optional as the Government Specification Content notes that this type of problem is for highly attaining pupils.

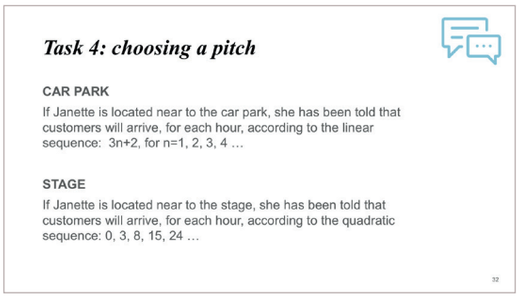
EXPLANATORY NOTES

Work through the slides so that the pupils are clear about what they are doing.

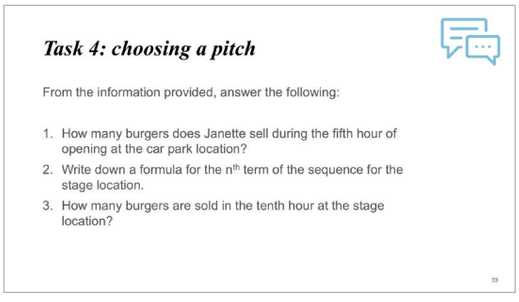
REFERENCE SLIDES



Slide 31



Slide 32



Slide 33

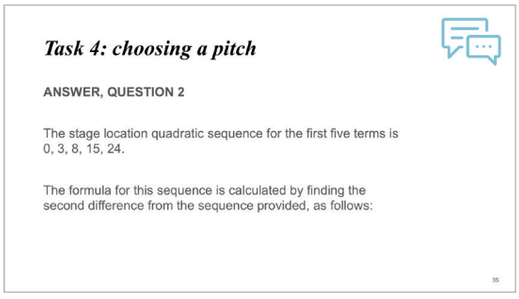
Task 4

PUPILS ARE ANSWERING TASK 4

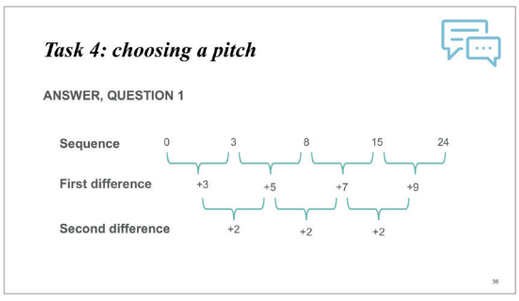
The answer to the problems are … (go through the slides)

REFERENCE SLIDES

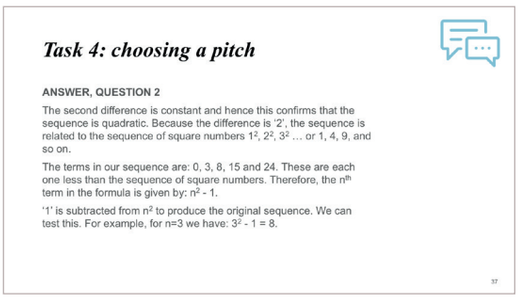
Slide 34



Slide 35



Slide 36



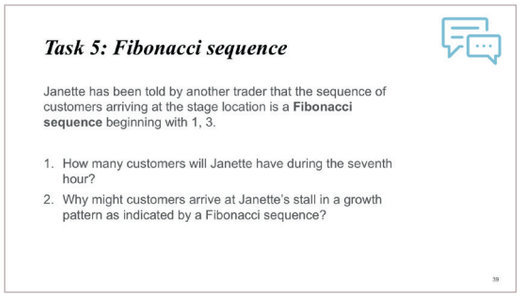
Slide 37

Slide 38

INTRODUCE AND MOTIVATE THE NEXT TASK

Again, this is another customer-arrival problem. Pupils may need an introduction to Fibonacci sequences if they are not familiar with them: in a Fibonacci sequence, the next number in the sequence is the sum of the previous two numbers.

REFERENCE SLIDE



Slide 39

Task 5

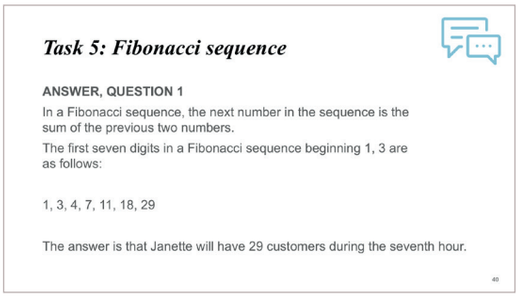
PUPILS ARE ANSWERING TASK 5

EXPLANATORY NOTES

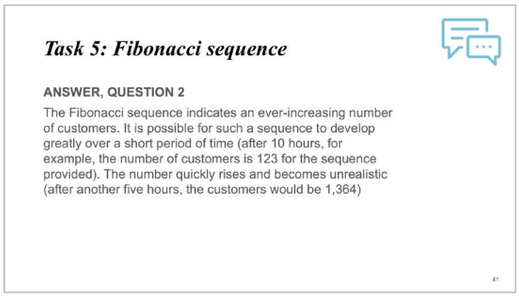
Question 2 is a hard question to answer and challenges pupils to think why the Fibonacci sequence might be appropriate. It might be worthwhile hinting that the more customers a business has, the more it is likely to attract new ones. The number of new customers could be related to those who have already visited the burger stall … That would be a reasonable thing to assume.

Also, note, however, that the Fibonacci sequence quickly produces very large numbers and hence the numbers would quickly become unrepresentative of the problem in hand. This point is important since, what we are doing, is trying to find a sequence of customers to the burger stall that will help predict demand for burgers. This will allow, if accurate, for Janette to plan accurately.

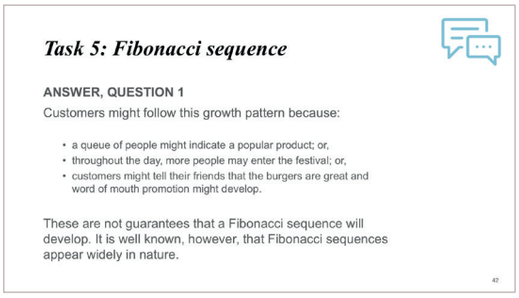
REFERENCE SLIDES



Slide 40



Slide 41



Slide 42

SOME FINAL THOUGHTS ON RISKS: DISCUSSION

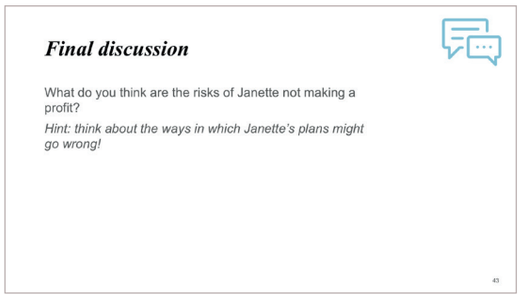
Risk is a very general term and is left deliberately undefined in the discussion. The idea is to get the pupils to begin to conceptualise risk (what can go wrong with Janette's plans for the festival).

The issue is important for studies later on (particularly in relation to thinking about probabilities) but also as a means of getting pupils to think about how risk might manifest itself in a particular situation.

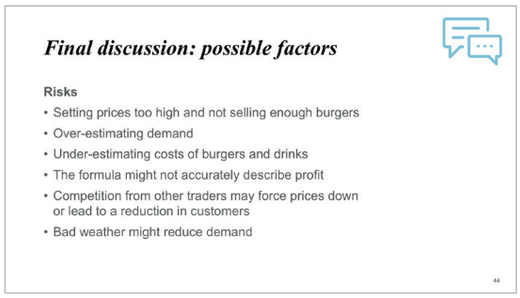
The basic ideas should be clear to pupils and they should be able to identify the risks in the slide with some encouragement:

- Setting prices too high and not selling enough burgers
- Over-estimating demand
- Under-estimating costs of burgers and drinks
- The formula might not accurately describe profit
- Competition from other traders may force prices down or lead to a reduction in customers
- Bad weather might reduce demand

REFERENCE SLIDES



Slide 43



Slide 44

Resources

1. LESSON PLAN (THIS DOCUMENT)
2. POWERPOINT
3. TASK HANDOUTS
4. SPREADSHEET

There are over 1.8m chartered accountants and students around the world – talented, ethical and committed professionals who use their expertise to ensure we have a successful and sustainable future.

Over 180,000 of these are ICAEW Chartered Accountants and students. We train, develop and support each one of them so that they have the knowledge and values to help build local and global economies that are sustainable, accountable and fair.

We've been at the heart of the accountancy profession since we were founded in 1880 to ensure trust in business. We share our knowledge and insight with governments, regulators and business leaders worldwide as we believe accountancy is a force for positive economic change across the world.

www.charteredaccountantsworldwide.com
www.globalaccountingalliance.com

ICAEW
Chartered Accountants' Hall
Moorgate Place
London
EC2R 6EA
UK

T +44 (0)20 7920 8100
E generalenquiries@icaew.com
icaew.com

Chartered Accountants Worldwide
Member

Global Accounting Alliance