

Linear Graphs

MATHEMATICS KEY STAGE 4

icaew.com/ICAEWmaths

Introduction

SUMMARY:

Use linear graphs to help understand the relationships between working and wages.

STRUCTURE:

The content is divided into two (approximately) 60 minute lessons.

PRESENTATION:

Italicised text are suggested scripts for the teacher to say. There are explanatory notes to aid quick understanding of some of the finance material.

GCSE ASSESSMENT OBJECTIVES ADDRESSED IN THE LESSON¹

The mathematical content specifications in this presentation are those used in the Mathematics GCSE Subject content and assessment objectives and is identified in red.

- A8: work with coordinates in all four quadrants
- A9: plot graphs of equations that correspond to straight-line graphs in the coordinate plane; use the form y = mx + c to identify parallel and perpendicular lines; find the equation of the line through two given points, or through one point with a given gradient
- **A10:** identify and interpret gradients and intercepts of linear functions graphically and algebraically
- A12: recognise, sketch and interpret graphs of linear functions, quadratic functions ...
- A14: plot and interpret graphs (including reciprocal graphs and exponential graphs) and graphs of non-standard functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration
- **A15:** calculate or estimate gradients of graphs and areas under graphs (including quadratic and other non-linear graphs), and interpret results in cases such as distance-time graphs, velocity-time graphs and graphs in financial contexts

¹ From government specification content where:

[•] All pupils will develop confidence and competence with the content identified by standard type

[•] All pupils will be assessed on the content identified by the standard and the <u>underlined</u> type; more highly attaining pupils will develop confidence and competence with all of this content

[•] Only the more highly attaining pupils will be assessed on the content identified by **bold** type. The highest attaining students will develop confidence and competence with the **bold** content.

Before lesson 1

HOMEWORK FOR THE PUPILS



Have a look at the following website which is from the Office for National Statistics:

https://visual.ons.gov.uk/the-gender-pay-gap-what-is-it-and-what-affects-it/

From the graphs on the page, answer the following questions:

- 1. What is the gender pay gap for full-time workers in 2016? Who is it in favour of, men or women?
- 2. What is the gender pay gap for part-time workers in 2016? Who is it in favour of, men or women?

Have a look at the graph on the website search for the section entitled 'Percentage of male and female employees in different occupational groups, UK, April to June 2016'

- 3. Which occupation has the largest difference, with more females being employed in it compared to males?
- 4. Does the occupation you identified have any explanation to offer for the gender pay gap?

FOR THE TEACHER

The two sets of homework - this is the first - will draw attention to websites from the Office for National Statistics (ONS) and, specifically, to their diagrammatic presentation of various investigations relating to pay in the UK. The essential idea is to get pupils used to reading and interpreting diagrams along with using and developing them in the following lessons. The first homework should be set a few days before the lesson begins. Clearly, it is assumed that the pupils have access to the internet either at home or at school.

The questions may look challenging but the answers are obvious once the diagrams are studied.

Lesson time



GENERAL NOTES

The associated powerpoint presentation has supporting script notes to help you. These can best be viewed by clicking 'View/Notes Page' in powerpoint. Items in *italics* are a proposed script for you to say. The content of the lesson plan follows closely the notes contained in the powerpoint.

Teaching and activities

CLASS DISCUSSION: INTRODUCE DISCUSSION ON THE HOMEWORK

Ask for volunteers to present or introduce their homework results to the class. (How this is tackled by the teacher will vary depending on how willing the class is to present their ideas). It may be preferable to have group ideas presented or to choose one or two pieces of homework for the teacher to present.

EXPLANATORY NOTES



A key skill being developed is in using financial websites confidently and in discovering that there is useful information and web content available.

HOMEWORK FOR THE PUPILS

the following work to be set before the lesson:

Have a look at the following website which is from the Office for National Statistics:

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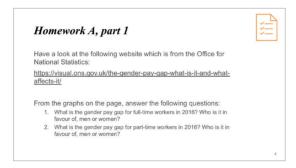
Have a look at the graph on the website and search for the section titled 'Percentage of male and female employees in different occupational groups, UK, April to June 2016'

- 3. Which occupation has the largest difference of more females being employed in it compared to males?
- 4. Does the occupation you identified have any explanation to offer for the gender pay gap?

POINTS FOR THE TEACHER TO WATCH OUT FOR

If possible, it would be helpful if the teacher shows the website during class to point out the answers to the homework. The last question is a discussion question: the website does not really offer clear links and that will be a teaching point about how far we can stretch the interpretation of diagrams presented to us.

Read through the slides and make sure that pupils understand how the different presentations reveal information i) over time and ii) between groups. Be sure to point out that the presentation should be chosen carefully to be informative.



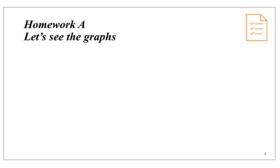
Homework A, part 2

Have a look at the graph on the website and search for the section entitled 'Percentage of male and female employees in different occupational groups, UK, April to June 2016'

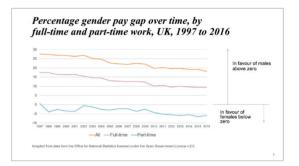
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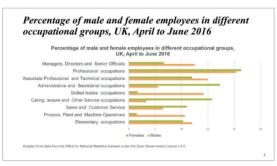
Slide 4



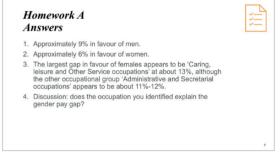
Slide 5



Slide 6



Slide 7



Slide 8

Slide 9

INTRODUCE THE CLASS TOPIC: LINEAR GRAPHS (WAGES AND WORKING)

The growth of jobs which are temporary, not fixed, and paid in relation to achievement (eg, number of deliveries made) has grown dramatically in recent years. It has altered how we think about working.

The growth of 'alternative' work is impacting on young people most and it is important to understand how earnings can vary with the different types of payment for work. Not all earnings are from 'employed' wages, as we will see.

There is a lot to think about and your mathematical skills can help us understand better how earnings vary with payment methods. In this lesson, we are going to use some graphical analysis to help.

Working – wages, salaries, bonuses and more! 1. Jordan, Mollie and Niamh are looking for their first jobs. They have all left school and are searching for summer work to fill the time before they all depart to some work abroad in October. 2. We are going to use linear graphs to understand some of the relationships between work and payment. But first, let's see how working and payment might work.

Slide 10

OPTIONAL SLIDES

Some of the following slides are optional depending on how deep you wish to go into the subject of different pay types. The shorter version focuses on pay types that young people are most likely to come across and which form the basis of this lesson.

Read through the slides

The basic graphical relationship between the payment base and earnings should slope upwards. That is, as more hours are worked, for example, we would expect earnings to increase.

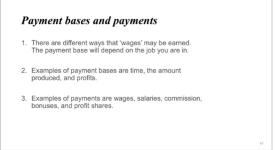
The details of how the line looks will vary depending on the detail of the relationship between the payment base in the rate of pay. Let's look at some of that detail.

The first payment method is hours worked. This can be in relation to an agreed, set number of hours for each week or there can be no hours guaranteed.

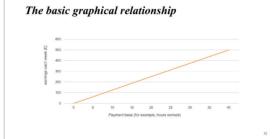
Your courier delivery driver or pizza delivery person is almost certainly on a piecework rate. Often such schemes generate low earnings although they don't have to.

Salaries are probably widely used. They are often used where the output from a job is hard to measure. The example shown is teachers. Other hard-to-measure jobs would be nurses, police officers and doctors.

REFERENCE SLIDES



CI: L 40



Slide 11 Slide 12

REFERENCE SLIDE (CONT)

Payment bases and payment in practice: time in the workplace (paid by the hour) Time in the workplace. The payment base is 'time' and can be paid as different rates: Basic Basic Overtime Example: Jo has agreed to work 35 hours each week where she is paid a basic rate of pay of \$7.50 per hour. Any hours over that are paid at an overtime rate at \$10 per hour. Jo works 40 hours one week and will be paid: 35 x £7.50 + 5 x £10 = £312.50. However, if Jo only works eight hours she will only be paid: 8 x £7.50 = £60.

Slide 13



Slide 15

Productivity or piecework

- Productivity is measured by job activity and can be measured by how much a person produces, delivers, etc. This is often referred to as a piece rate. Let's see how the acquired to account of the country of the product of the country of the countr
- Suppose that £0.50 is paid by the pizza company for each delivery made.
 Assume, on average, that a tip of £1 is received for each delivery. On this basis ti might be expected that £1.50 is earned for each delivery. How much might be earned for a 40-hour week when five deliveries are made each hour? The

£1.50 x 40 x 5 deliveries = £300

Slide 14

THE FOLLOWING ARE OPTIONAL SLIDES THAT SHOW FURTHER, ALTERNATIVE WAYS OF EARNING MONEY IN WORK

Commission-based work is often an entry-level employment for those developing careers in sales and/or marketing. It is a way of incentivising individuals to meet sales targets. There are some jobs which are commission-only. Would you want a job like that?

The underlying principles in terms of how pay is calculated for bonuses and performance-related pay is essentially the same as commission. The difference is that under commission, the payment base is sales whereas under bonuses or performance-related pay it will be another base, almost certainly production related.

Profit shares are usually thought of in terms of executive power. However, the John Lewis stores group has famously had a profit share arrangement for many years where all employees take part.

Commission

- How much a person sells. This is called 'commission' and it is paid in relation to sales. Commission is often paid on top of a basic salary. The payment base is sales. This can be calculated in a number of ways: for example, as an amount
- Example: a person selling confectionery to local shops may earn a basic salary
 of £15,000 a year. In addition, they may be paid an additional 2% of the value of
 the sales made. A person selling £25,000 of confectionery a year would then
 have earnings of:

£15,000 + (2/100) x £25,000 = £15,500

Slide 16

Profit share

- This is like performance-related pay but it is not based on a specific criterion that
 can be specifically related to the individual. It reflects a general, team-based
 reward for working profitably, John Lewis department store has a well-known
 profit share scheme in which all stell fixes part.
- Example: a senior executive may have a salary of £50,000 and be paid an additional 0.5% of the profits earned by the business she works for. If the business makes a profit of £1.2m then her earnings for the year would be:

£50,000 + (0.5/100) x £1,200,000 = £56,000.

Slide 18

Task 1

CLASS PROBLEM: INTRODUCE TASK 1

Present task 1 for the pupils to answer

The purpose of this task is to get pupils to think about the implications of choosing one payment method over another. Of course, when the time comes they may not have a choice but the implications of the differences between them should be drawn out. There may be others in addition to those identified in the answers.

Have a go at this task. Think about what work entails which payment method and try to think about the benefits of payment methods and also some of the risks. For example, it is often risky to choose a job where earnings are not guaranteed because that means it is possible you may not get the earnings you need in order to pay the bills you have to.

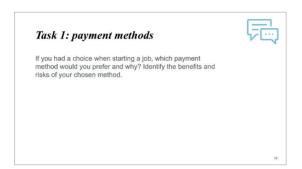
Bonuses and performance-related pay

- Here, wages vary directly with a defined work-based criterion. The payment base is a pre-set performance criterion and is paid on top of a basic salary. It can
- Example: a production worker has a basic salary of £25,000 and is paid a
 performance-related bonus if she produces 8,000 metal castings a month. Th
 bonus is £100 for each month she meets her target. If her production target is
 met in 8 months out of 12 then her annual earnings would be:

£25,000 + 8 x £100 = £25,800.

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Slide 19



PUPILS ARE ANSWERING TASK 1



EXPLANATORY NOTES

The slide is self explanatory and it may be useful to add further illustrations of the example payment methods and their benefits and risks.

REFERENCE SLIDE



Slide 21



EXPLANATORY NOTES

(THIS IS AN ALTERNATE TO THE PREVIOUS SLIDE WHICH OFFERS A WIDER RANGE OF PAYMENT METHODS TO CONSIDER) The slide is self explanatory and it may be useful to add further illustrations of the example payment methods and their benefits and risks





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Slide 23

INTRODUCTORY TEACHING

The scene involves analysing the payment methods of two jobs. The difference between them is that Job 1 offers a basic salary. Both jobs provide hourly rates of pay. The hourly rate is provided to indicate that there would be a variable level of earnings depending on how many hours are worked. However, while Job 1 has a minimum earnings level of £57.50, Job 2 has a minimum of £zero. There is no safety net with Job 2.

REFERENCE SLIDE



Slide 24

Task 2

CLASS PROBLEM: INTRODUCE TASK 2

There are quite a few tasks in task 2 although, in reality, the steps have been broken down so that there should be a clear sense of direction for pupils.

It is left up to the teacher to indicate if the pupils should pause after completing a set number of tasks before proceeding with the final set. The following dialogue reflects this:

Option 1

Have a go at these tasks. Work in your pairs. Use the graph paper provided.

Option 2

Have a go at questions 1 and 2 and we will then look at the answers. Work in your pairs. Use the graph paper provided.

Now have a go at questions 3 to 8 and then we will look at the answers.

Now have a go at question 9. In question 9 think broadly about the non-financial factors that might influence which job you would prefer.

REFERENCE SLIDES





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PUPILS ARE ANSWERING TASK 2

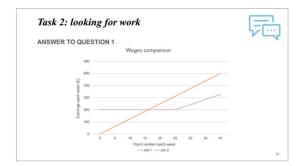


EXPLANATORY NOTES

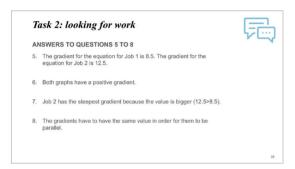
This should be straightforward for pupils to complete. Note that they may have different scales on the axes.

Note also that the line for Job 1 in the range 0-25 hours should not be interpreted that a person could work less than 25 hours and still be paid the same amount. It simply records the fact that wages are the same over the 0-25 hour range and that the related line is flat over that range. It starts increasing after 25 hours have been worked.

The diagrams are fully explanatory. Please teach from them.



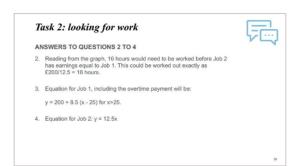
Slide 27



Slide 29



Slide 31



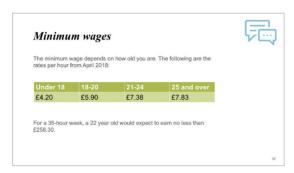
Slide 28



Slide 30

TEACHING: MINIMUM WAGES

This slide is for information only. It would be interesting to see if it causes any discussion about the level of pay that some people earn.



Slide 32

Homework for lesson 2

We've now got a fairly good understanding of how earnings are related to what we do in our job and the base - payment base - on which wages are calculated. Let's have a look at some homework using graphs on some real data.

Homework: Ask the pupils to complete the homework. This is included in the Tasks folder to print off if necessary.

Have a look at the website:

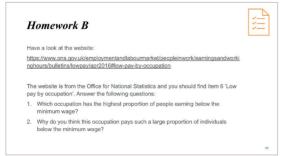
https://www.ons.gov.uk/employmentandlabourmarket/peopleinwork/earningsandworkinghours/bulletins/lowpay/apr2016#low-pay-by-occupation

The website is from the Office for National Statistics and you should have found item 6 'Low pay by occupation'. Answer the following questions:

- 1. Which occupation has the highest proportion of people earning below the minimum wage?
- 2. Why do you think this occupation pays such a large proportion of individuals below the minimum wage?

REFERENCE SLIDES

Slide 33







Lesson 2

General notes: The associated powerpoint presentation has supporting script notes to help you. These can best be viewed by clicking 'View/Notes Page' in powerpoint. Items in italics are a proposed script for you to say. The content of the lesson plan follows closely the notes contained in the powerpoint.

Teaching activities

CLASS DISCUSSION: INTRODUCE DISCUSSION ON THE HOMEWORK



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HOMEWORK

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Have a look at the website:

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EXPLANATORY NOTE

The slides contain basic information. Some explanation might be needed on 'Elementary occupations'.

An 'Elementary' occupation is:

One which 'will usually require a minimum general level of education (ie, that which is acquired by the end of the period of compulsory education). Some occupations at this level will also have short periods of work-related training in areas such as health and safety, food hygiene, and customer service requirements.'

ONS Standard Occupational Classification 2010 volume 1 Structure and description of unit groups.

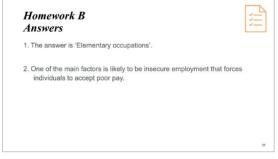
The answer to Question 2 might act as discussion point. The key point is for pupils to undertake comparisons, using the graph of other groups paying below the minimum wage. For example, there are few below-minimum wage earners in professional occupations.

The answer provided is likely to be true but there will also certainly be other factors. It might be worthwhile asking the class to identify what might be other factors that perhaps force people into very low jobs. There is no hard and fast answer, here; it is only to raise awareness of the issue and to let the pupils themselves present some ideas.

REFERENCE SLIDES



Slide 34



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Homework B You were asked to do the following: Have a look at the website: https://www.ons.gov.uk/employmentandlabourmarket/beopleinwork/earningsandworkinghours/bulletins/lowpay/apr/2016#low-pay-by-occupation The website is from the Office for National Statistics and you should find item 6 'Low pay by occupation'. Answer the following questions: 1. Which occupation has the highest proportion of people earning below the minimum wage? 2. Why do you think this occupation pays such a large proportion of individuals below the minimum wage?

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INTRODUCE AND MOTIVATE THE NEXT TASK

On the farm, three grades of strawberries are grown and they are sold at different prices to the supermarkets depending on their quality. Which strawberries are grown depends on the condition of the soil in the field and what methods of fertilisation are used.

Go through the slides to ensure that pupils are clear about what to do.

EXPLANATORY NOTES



Question 1: The tables are completed by setting one of the variables in turn to zero and then solving for the remaining unknowns. For example, in the table for field 1, when B is set to zero then the value of P can be found from the following sequence:

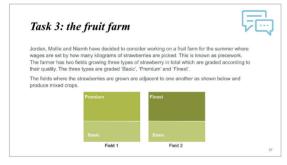
Field 1: 0.35B + 0.5P = £12.00 per hour.

Set B to zero gives $0.35 \times 0 + 0.5P = £12.00$ per hour.

P is then clearly equal to 12/0.5 = 24.

It might be worth completing this example to the class to get them started on the problem.

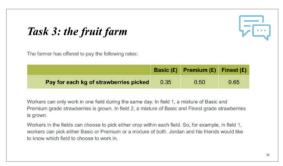
REFERENCE SLIDES



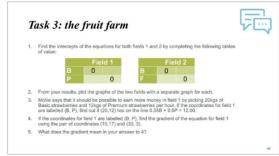
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Slide 38



Slide 40

Task 3



PUPILS ARE ANSWERING TASK 2

The answer to the problems are ... (go through the slides)

Task 3: problem 1

The answers are derived as follows:

Field 1: 0.35B + 0.5P = £12.00 per hour. Set B to zero gives $0.35 \times 0 + 0.5P = £12.00$ per hour.

P is then clearly equal to 12/0.5 = 24.

Field 1: 0.35B + 0.5P = £12.00 per hour. Set P to zero gives $0.35B + 0.5 \times 0 = £12.00$ per hour. B is then clearly equal to 12/0.35 = 34.28.

Field 2: 0.35B + 0.65F = £11.10 per hour. Set B to zero gives $0.35 \times 0 + 0.65F = £11.10$ per hour. F is then clearly equal to 11.1/0.65 = 17.08.

Field 2: 0.35B + 0.65F = £11.10 per hour. Set F to zero gives $0.35B + 0.65 \times 0 = £11.10$ per hour. B is then clearly equal to 11.1/0.35 = 31.72.

Task 3: problem 2

The diagram uses the data from the previous table to find the coordinates on the axes and to plot a straight line between them.

Task 3: problem 3

This can be answered by plotting the coordinates on the graph or by seeing if the values that Mollie proposes, when inserted into the equation for field 1, actually add up to 12. As can be seen, they don't.

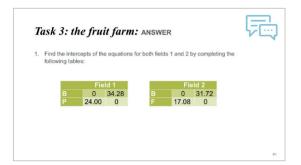
Task 3: problem 4

Teacher to go through this. The instruction will depend on how the pupils have been taught to find gradients.

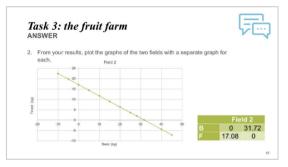
Note that the gradient is negative which means it is sloping downwards. The diagram drawn for field 1 is sloping downwards and so our answer makes sense!

Task 3: problem 5

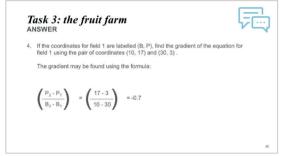
Teach from the slide



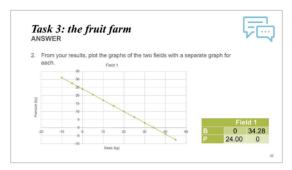
Slide 41



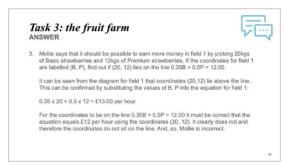
Slide 43



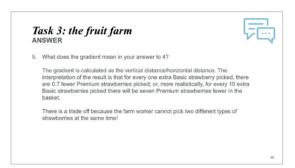
Slide 45



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SOME FINAL THOUGHTS

This is about zero hours and tries to present some thoughts about them.

What happens in practice

Most employees under such contracts have fairly regular hours (but, remember, not guaranteed) and so can expect a regular income. But there will inevitably be some employees who will face a very irregular working life.

Why would an employer offer zero hours?

Generally, it allows the employer to reduce labour costs to a minimum level and avoid the situation where an employer is paying a worker - on a regular-hour contract - when they are idle (which can sometimes happen when business activity fluctuates). This reduction in paying for 'idle time', in effect, transfers the risk of being idle to the employee from the employer: this is because when there is no work to do the worker under zero-hour contract is not paid.

Resources

- 1. LESSON PLAN (THIS DOCUMENT)
- 2. POWERPOINT
- 3. TASK HANDOUTS
- 4. SPREADSHEET

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